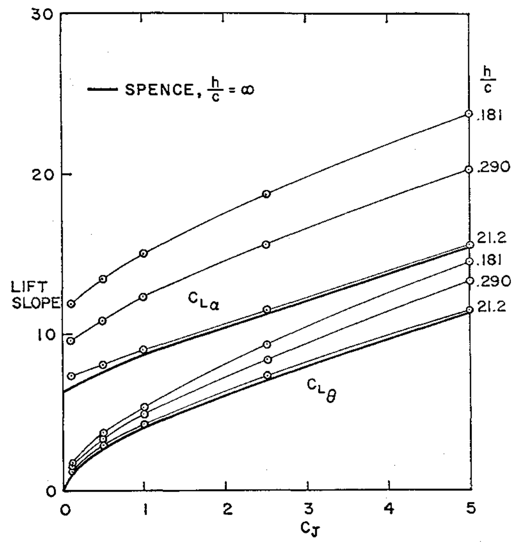

Fig. 3 Lift curves in ground effect.

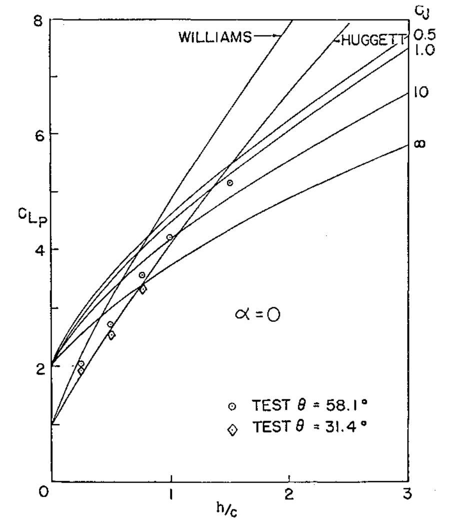

Fig. 4 Wake blockage.

may be transformed into an integral over the jet wake and is thus called the wake lift component C_{Lw}. This is evaluated numerically and is plotted in Fig. 2. We find that C_{Lw} is bounded for $C_J = \infty$ (shown in Fig. 2 as "Lift Limit"). However, for the case of $G \to 0$ and C_J finite we get $C_{Lw} \to 0$.

Thus, we express the singular blowing lift slope as

$$C_{L_\theta} = C_{LN} + C_J + C_{Lw} \tag{3}$$

By using Eq. (2), C_{L_α} may be determined, and is shown in Fig. 3. As a limiting case check, the results of Spence[1] for $h/c = \infty$ are shown.

Interesting results occur for extreme C_J, G values. Pressure lift on the airfoil, $C_{LP} = C_{LN} + C_{Lw}$, is most significant, since the jet momentum lift C_J is unaffected by the ground plane. For large values of h/c and strong blowing, $C_{LN} \sim O(C_J^{1/2}/G)$, $C_{Lw} \sim O(1)$. Thus a dominant part of the lift is carried near the nose. For weak blowing,[3] we get $C_{LN} \sim O(C_J^{1/2}/G)$, $C_{Lw} \sim O(C_J^{1/2})$. Again, for low values of h/c and finite blowing we get $C_{LN} \sim O(C_J^{1/2}/G)$, $C_{Lw} \to 0$. Other results may be obtained similarly, giving an engineering insight into the behavior of the lifting terms. Frequently the nose flow is the critical factor, since high negative pressures and adverse gradients cause both compressible and viscous effects. This may be controlled by detail design of nose radius and droop, involving thickness and camber effects for which a procedure is given in Ref. 3.

Wake Blockage

This phenomenon is important since it may make it impossible to achieve the theoretical lift. Experiments by Huggett[4] showed that blockage caused a behavior analogous to conventional stall, where dC_L/dC_J abruptly reduces and the lift on the airfoil reaches a maximum. This is caused by the jet impinging on the ground and effectively blocking the lower surface flow. For a given h/c, the pressure lift (C_{L_p}) remains constant for all C_J above a critical value. Tests showed this critical C_{L_p} was independent of θ ,for $\alpha = 0$.

Theoretical estimates of the maximum C_{L_p} are made by Williams[3] and Huggett.[4] Details of both models can be criticized. An independent estimate of using the present linear theory gives a formula for blockage as $C_{L_p} = (2h/c)^{1/2} \{4\pi^{-1/2}/G + C_{L_w}C_J^{-1/2}\}$. This curve has similar characteristics to those of Ref. 4, although it too is certainly not a precise model of the flow. The three curves are shown in Fig. 4, with some data.[4] The tests of Ref. 4 were with a fixed ground although these are improper boundary conditions.

Evidently a true test involves removing the ground boundary layer, by suction or a moving belt. This might be expected to increase the value of blockage C_{L_p}. None of the three blockage theories takes explicit account of viscosity. Consequently, both the theoretical and experimental values of Fig. 4 should be regarded as very approximate; however, in practical design situations blockage will be a major consideration.

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Redundant Analysis of a Group of Airframe Problems

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A LARGE number of the mathematical models prepared for discrete-element redundant analysis of airplane structural problems at the Lockheed-Georgia Company since November 1961 were reviewed recently. A comparison was made of the maximum number of simultaneous equations to be solved by application of the force and the displacement methods of analysis for compatibility or equilibrium, respectively. The results of the review are shown in Table 1 as the number of redundants n for the force method and the degrees of freedom d for the displacement method.

The tabulation shows that the maximum number of simultaneous equations to be solved for displacement-method application is three to five times more than for the force-method application to the practical airframe design engineering problems reviewed. Table 1 is representative of the redundant-analysis problem solutions performed for the C-141 and C-5A airplanes. All of the mathematical models listed in Table 1 were solved by the particular version of the force method, which is described in Ref. 1. Computations were performed on IBM 7094 computers via the flutter and matrix algebra

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Table 1 Maximum number of redundants and degrees of freedom

Problem title	Total structural elements E	Gridpoints G	Restraints and supports R	Maximum number of simultaneous equations without problem subdivision: Force method, number of redundants n = E + R − 3G	Displacement method, degrees of freedom d = 3G − R	Ratio d/n	Number of modules (subdivisions)
1. C-141 problems							
a. Wing-fuselage intersection	1343	358	28	297	1046	3.52	-
b. Vertical stabilizer	266	89	47	46	220	4.78	-
c. Horizontal stabilizer	371	108	14	61	310	5.07	-
d. Fuselage shell and cargo floor	1764	714	706	328	1436	4.39	-
e. Aft fuselage and fin	1798	602	402	394	1404	3.57	-
2. Preliminary stage C-5A problems							
a. Wing (symmetrical)	1982	536	66	440	1542	3.51	-
b. Wing (anti-symmetrical)	1967	554	121	426	1541	3.62	-
c. Aft barrel (single shell and floor)	1920	506	91	493	1427	2.89	-
d. Aft barrel (double shell and floor)	1876	508	137	489	1387	2.83	-
e. Wing-fuselage intersection (initial version)	2211	667	213	423	1788	4.22	-
f. Wing-fuselage intersection (second version)	4172	1285	472	789	3383	4.29	2
3. Production stage C-5A problems							
a. Wing	4117	1309	567	757	3360	4.44	9
b. Aft fuselage shell and cargo floor	4049	1215	687	1085	2958	2.72	7
c. Wing-fuselage intersection	4533	1242	216	1023	3510	3.42	3
d. Aft fuselage and fin	9414	2867	944	1757	7657	4.36	12
e. Forward fuselage shell and cargo floor	2877	884	513	738	2139	2.90	6

system (FAMAS) complex of programs, using single precision. A different combination of FAMAS programs has also been used for solution of a few small problems (about 160 redundants with 900 degrees of freedom) by the displacement method.

It is important to note that the Table 1 compilation is expressed in terms of total problem size. Matrix coupling or partitioning schemes were not used except as indicated by the "number of modules (subdivisions)" column. Such schemes were used very little at Lockheed-Georgia until the summer of 1965, when the larger problem sizes associated with the C-5A (see Table 1) made problem subdivision essential because of matrix-solution accuracy requirements and excessive computer-time usage. In the coupling schemes that have been used extensively since then, the mathematical model of the structure is subdivided into several modules for solution of the set of equations for the redundants in each module; then compatibility of deformations is established at all module boundaries, and the effects are propagated to complete the solution of the total problem. Subdivision and coupling of a large mathematical model in this fashion reduces the total number of equations to be solved simultaneously into a series of smaller sets of equations, one set per module, and improves accuracy of solution, reduces computer time, and facilitates maximum concentration of manpower on a given problem solution. However, these advantages must be balanced against the additional engineering preparation and more numerous matrix operations which are required. Spring rates at all module boundaries must be determined, provision must be made for external load feedback across all module boundaries, and compatibility established throughout the entire model. Although this procedure is simple in concept, it becomes an error-prone and time-consuming operation when implemented on a large and complex mathematical model subdivided into as many as 12 modules.

The same logic described here for subdividing a mathematical model of a complex structure into several coupled modules for force-method solution can also be applied for displacement-method solution. However, the much higher number of simultaneous equations associated with displacement-method formulation (degrees of freedom) than with force-method formulation (number of redundants) prior to model subdivision is a very significant handicap for the larger problems, especially the intersection models. Prior to subdivision, model 2.f. (Table 1) had 789 redundants and 3383 degrees of freedom; model 3.c., 1023 redundants and 3510 degrees of freedom; and model 3.d., 1757 redundants and 7657 degrees of freedom. The engineer's freedom of action in subdividing an intersection model into constituent modules is usually limited severely by the large size and complexity of the structural intersection region. For instance, model 3.d. (Table 1) as expressed in terms of 1757 simultaneous equations (for redundants) is a far less formidable problem than when expressed in terms of 7657 simultaneous equations (degrees of freedom). Solution of the 7657 equations required for displacement-method application would constitute a practically insuperable problem in view of the structural complexity of the intersection region, the large number of modules into which the problem would have to be subdivided, and the sheer massiveness of the computation and bookkeeping required.

It should be recognized that the information accumulated in Table 1 and the related discussion concern typical redundant airframe structural problems. Generalization to other types of structures is not necessarily applicable.

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